Pohozhaev Identities and Applications

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Beginning from the seminal Pohodzaev paper (1965), the Pohodzaev Identities, first developed for the Dirichlet problem for elliptic quasilinear equations, have been extended to study many cases for different PDE classes of equations, even for some fractional PDE's.

The present talk I will focus on the extension of Pohodzaev approach in the last 25 years for its applications to some mixed elliptic-hyperbolic quasilinear equations. This study began from the series of results of Lupo - Payne, where using some symmetries in the linear homogeneous case, the dilation invariance law is strongly related to certain Pohodzaev type identities that give examples of critical exponent phenomena. Let us mention here the big difference with elliptic case where the work is with classical smooth solutions, for whom the situation is: in the case of supercritical power we have the nonexistence of nontrivial solutions in so called "starshaped domains", and in the opposite case of subcritical power there exist some nontrivial solutions. Now, for mixed type equations the statement of the problems in both cases are in some special functional weight classes of "generalized solutions". The reason is the singularity which usually naturally exists in the solution even in some classical domains. I will give some example when the generalized solution in the linear case has an exponential growth at only one point of singularity. The singularity does not propagate along the characteristic light cone. Some open problems will be formulated and discussed.

Keywords: Pohozhaev identities, critical exponent, supercritical cases, nonexistence of nontrivial solutions